Preprint RM3-TH/02-17

Proton and neutron polarized structure functions from low to high Q^2 [a]

S. Simula(*), M. Osipenko(**), G. Ricco(***) and M. Taiuti(***)

(*)Istituto Nazionale di Fisica Nucleare, Sezione Roma III,
Via della Vasca Navale 84, I-00146 Roma, Italy
(**)Physics Department, Moscow State University, 119899 Moscow, Russia
(***)Dipartimento di Fisica, Universitá di Genova and INFN, Sezione di Genova,
Via Dodecanneso 33, I-16146, Genova, Italy

Abstract

Phenomenological parameterizations of proton and neutron polarized structure functions, g_1^p and g_1^n, are developed for $x \gtrsim 0.02$ using deep inelastic data up to $\sim 50\ (GeV/c)^2$ as well as available experimental results on photo- and electro-production of nucleon resonances. The generalized Drell-Hearn-Gerasimov sum rules are predicted from low to high values of Q^2 and compared with proton and neutron data. Furthermore, the main results of the power correction analysis carried out on the Q^2-behavior of the polarized proton Nachtmann moments, evaluated using our parameterization of g_1^p, are briefly summarized.

[a] To appear in the Proceedings of the II International Symposium on the *Gerasimov-Drell-Hearn sum rule and the spin structure of the nucleon*, Genova (Italy), July 3-6, 2002, World Scientific Publishing (Singapore), in press.

1 Introduction

The experimental investigation of lepton deep-inelastic scattering (DIS) off proton and deuteron targets has provided a wealth of information on parton distributions in the nucleon. In the past few years some selected issues in the kinematical regions corresponding to large values of the Bjorken variable x have attracted a lot of theoretical and phenomenological interest; among them one should mention the occurrence of power corrections associated to *dynamical* higher-twist operators measuring the correlations among partons. The extraction of the latter is of particular relevance since the comparison with theoretical predictions either based on lattice QCD simulations or obtained from models of the nucleon structure represents an important test of QCD in its non-perturbative regime.

In Refs. [1] and [2] phenomenological fits of the world data on the unpolarized nucleon structure functions F_2^N and F_L^N were used to evaluate Nachtmann moments and power correction analyses were carried out. In Ref. [3] the same twist analysis was extended to the polarized proton case by developing a parameterization of g_1^p, which describes the DIS proton data up to $Q^2 \sim 50\ (GeV/c)^2$ and includes a phenomenological Breit-Wigner ansätz able to reproduce the existing electroproduction data in the proton-resonance regions. The interpolation formula for g_1^p was successfully extended down to the photon point, showing that it nicely reproduces the Mainz data [4] on the energy dependence of the asymmetry of the transverse photoproduction cross section.

The plan of this contribution is as follows. In Section 2 we extend our phenomenological parameterization of Ref. [3] to the neutron polarized structure function g_1^n. In Section 3 the generalized Drell-Hearn-Gerasimov (DHG) sum rules are predicted from low to high values of Q^2 and compared with proton and neutron data. Finally, in Section 4 the main results of the power correction analysis carried out in Ref. [3] on the Q^2-behavior of the polarized proton Nachtmann moments will be briefly summarized.

2 Phenomenological parameterizations of g_1^p and g_1^n from low to high Q^2

Following Ref. [3] we write the polarized nucleon structure functions as the sum of three contributions

$$g_i(x, Q^2) = g_i^{(el.)}(x, Q^2) + g_i^{(res.)}(x, Q^2) + g_i^{(non-res.)}(x, Q^2) \qquad (1)$$

where the suffix p or n is omitted for simplicity, $g_i^{(el.)}$, $g_i^{(res.)}$ and $g_i^{(non-res.)}$ are the elastic, resonant and non-resonant contributions to g_i, respectively, and $i = 1, 2$. In Eq. (1) possible interference terms between the resonant and non-resonant contributions are neglected, since they are well beyond the scope of our phenomenological fit.

The elastic contribution is the simplest one, because it can be expressed in terms of the nucleon Sachs form factors (see Eqs. (17-18) of Ref. [3]). As for the non-resonant terms

$g_i^{(non-res.)}$ we adopt the following decomposition

$$g_1^{(non-res.)}(x,Q^2) = g^{\Delta\sigma}(x,Q^2) + \frac{4M^2x^2}{Q^2} g^{LT}(x,Q^2)$$
$$g_2^{(non-res.)}(x,Q^2) = -g^{\Delta\sigma}(x,Q^2) + g^{LT}(x,Q^2) \quad (2)$$

where $g^{\Delta\sigma}$ is the contribution arising from the transverse asymmetry A_1, while g^{LT} is the LT interference governing the asymmetry A_2. In Ref. [3] we have developed an interpolation formula for $g^{\Delta\sigma}$ of the form

$$g^{\Delta\sigma} = \frac{W^2 - M^2}{2W^2} \sum_{j=1}^{N} a_j \left[1 + \frac{W^2}{Q^2 + Q_R^2}\right]^{\alpha_j} \left[\frac{W^2 - W_\pi^2}{W^2 - W_\pi^2 + Q^2 + W_T^2}\right]^{\beta_j} \quad (3)$$

where W_π is the pion production threshold and t is a parameter aimed at describing the logarithmic scaling violations in the DIS regime, namely: $t = \ln\left\{\ln\left[(Q^2+Q_0^2)/\Lambda^2\right]/\ln\left(Q_0^2/\Lambda^2\right)\right\}$. In Eq. (3) the parameter Q_R^2 describes the transition from the expected dominance of the Regge behavior at $Q^2 \lesssim Q_R^2$ to the partonic regime at $Q^2 >> Q_R^2$, while the quantities a_j, α_j and β_j are parameters assumed to depend linearly on t. Finally, the term g^{LT}, contributing to Eq. (2), is parameterized as in Eq. (26) of Ref. [3].

All the parameters appearing in Eq. (3) but W_T can be determined by fitting existing measurements of the asymmetry A_1 in the DIS kinematics ($W \gtrsim 2\ GeV$). As explained in Ref. [3] the value of W_T can be fixed by requiring the reproduction of the DHG sum rule. We point out that in Eq. (3) $g^{\Delta\sigma}$ is assumed to behave in the Bjorken limit as a power of x at low x. There is no strong argument in favor of such an assumption and therefore Eq. (3) has to be considered as a simple approximation valid in a limited x-range. In this respect, since existing data for both proton and neutron targets are scarce below $x \sim 0.02$, we consider $x \gtrsim 0.02$ as the x-range of applicability of our parameterization (3). This implies that we cannot check the Bjorken sum rule, because the latter is extremely sensitive to the behavior of g_1^n at very low x (below 10^{-2}). Therefore, in this contribution the parameterization of g_1^p is directly taken from Ref. [3], and the theoretical value of the Bjorken sum rule is inserted in the fitting data set for g_1^n.

In case of proton we used [3] 14 parameters against 209 experimental points, obtaining for the χ^2 variable (divided by the number of *d.o.f.*) the minimum value of 0.66. Repeating the same procedure in case of the neutron (with the addition of the Bjorken sum rule as a constraint) we have got $\chi^2 = 0.90$ against the 245 experimental points from Refs. [5, 6]. We anticipate that the value of the parameter W_T, fixed through the DHG sum rule, turns out to be $W_T = 0.475$ (0.451) GeV in case of proton (neutron).

In the resonance regions ($W \lesssim 2\ GeV$) we adopt a simple Breit-Wigner shape to describe the W-dependence of the contribution of an isolated resonance R, while its Q^2-dependence can be conveniently expressed in terms of the helicity amplitudes $A_{1/2}^R$, $A_{3/2}^R$ and $S_{1/2}^R$. The explicit expression for $g_1^{(res.)}$ and $g_2^{(res.)}$ are given by Eqs. (29-35) of Ref. [3]. We have considered all the "four-star" resonances of the PDG [7] having a mass $M_R < 2\ GeV$ and a

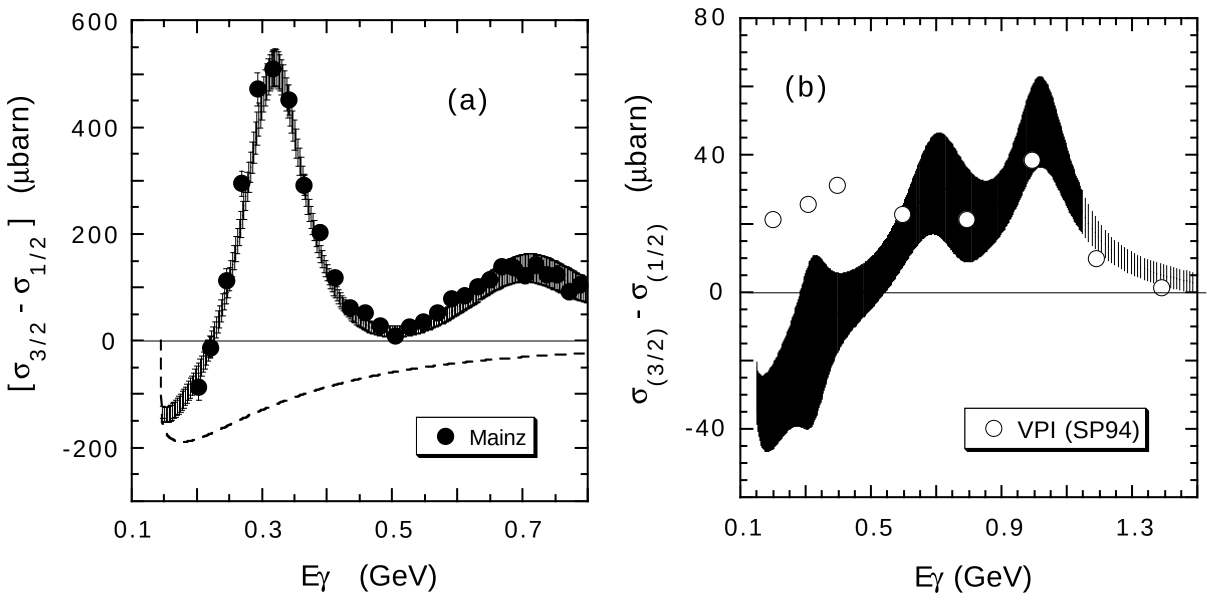

Figure 1: Asymmetry of the transverse photoabsorption cross section for the proton (a) and for the difference between proton and neutron (b) versus the photon energy E_γ. Full dots are the data from Ref. [4], while open dots are the results of the VPI multipole analysis labelled $SP94$ (see Ref. [8]). The shaded area is our prediction, while the dashed line in (a) is our non-resonant contribution, which is sensitive to the value of the parameter W_T (see text). Fig. 1(a) is adapted from Ref. [3].

total transverse photoamplitude $\sqrt{|A^R_{1/2}|^2 + |A^R_{3/2}|^2}$ larger than 0.050 (0.040) $GeV^{-1/2}$ in case of proton (neutron). Our final results at $Q^2 = 0$ are reported in Fig. 1. It can be seen that they positively compare with all the Mainz data [4] in case of the proton and with the results of the VPI multipole analysis [8] for photon energies E_γ above ≈ 0.5 GeV in case of the isovector-isoscalar (VS) channel. There is however a clear discrepancy when $E_\gamma \lesssim 0.5$ GeV. It should be reminded that the results of multipole analyses are consistent with the VV part of the DHG sum rule, while they differ remarkably in case of the VS part.

3 Generalized DHG sum rules from low to high Q^2

In this Section we present our predictions for the generalized DHG sum rules based on the parameterizations of g_1^p and g_1^n described previously.

We have calculated the inelastic part of the first moment of g_1, defined as $\Gamma_1(Q^2) \equiv \int_0^{x_\pi} dx\ g_1(x, Q^2)$, where x_π is the pion threshold. Our results are shown in Fig. 2 and compared with both DIS data and the new $JLab$ data [9] which cover the intermediate Q^2-region ranging from ≈ 0.2 to ≈ 1 $(GeV/c)^2$. It can be seen that our results nicely fit the data in the DIS kinematics and agree at very low values of Q^2 with the results of Heavy Baryon Chiral Perturbation Theory ($HB\chi pT$) obtained in Ref. [11]. However, while the proton $JLab$

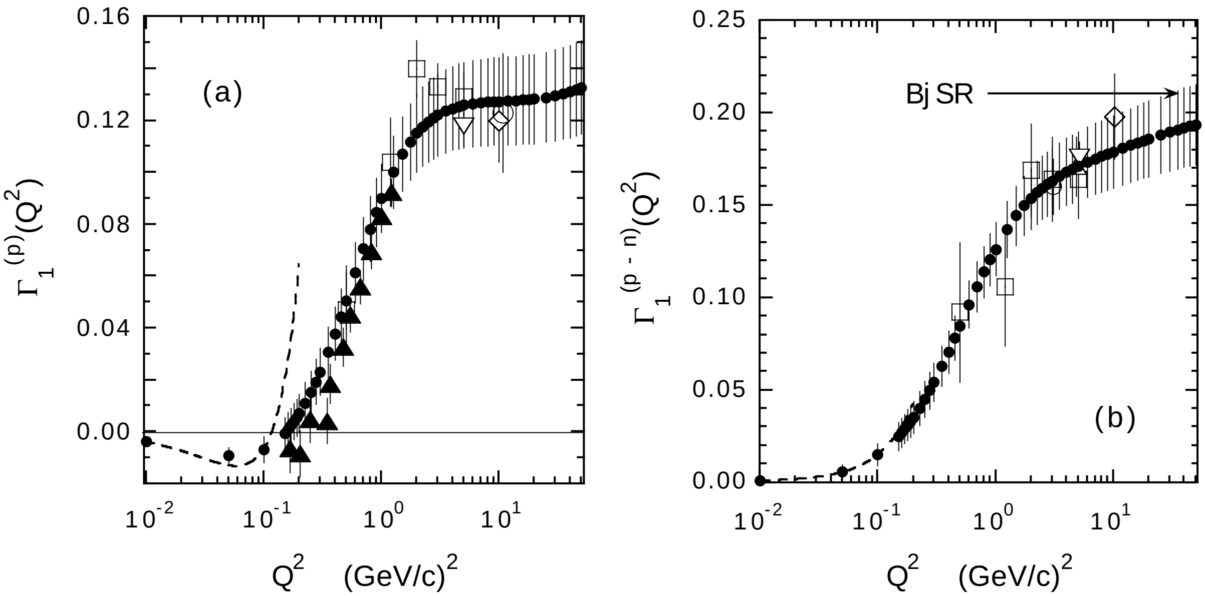

Figure 2: Inelastic part of the first moment of g_1^p (a) and of $[g_1^p - g_1^n]$ (b). Full dots represent our results. In (a) full triangles are data from $JLab$ [9], while open squares, diamonds, dots and triangles are from Ref. [10](a,b,c,d), respectively. In (b) open diamonds, squares and reverse triangles are from Ref. [5](a,b,c), while open dots and triangles are from Ref. [6](a,b), respectively. Dashed lines are the $HB\chi pT$ predictions of Ref. [11].

data change sign at $Q^2 \simeq 0.25$ $(GeV/c)^2$, our parameterization of g_1^p predicts the occurrence of the zero-crossing point at $Q^2 = 0.16 \pm 0.04$ $(GeV/c)^2$. Moreover the applicability of the $HB\chi pT$ to the nucleon has been criticized in Ref. [12], where important finite-mass effects have been found.

The resonance contribution to the transverse cross section asymmetry of the neutron, defined as $I_t^{(n)}(Q^2) \equiv \int_{\nu_\pi}^{\nu_{max}} d\nu \ (\sigma_{1/2} - \sigma_{3/2})/\nu$ with ν_{max} corresponding to $W_{max} \simeq 2\ GeV$, has been recently determined by the $JLab$ experiment $E94010$ [13]. Our results are reported in Fig. 3 as full dots and compared with the $JLab$ data. It can be seen that a striking discrepancy occurs below $Q^2 \simeq 0.4$ $(GeV/c)^2$. A similar discrepancy is shared also by the result of the unitary isobar model of Ref. [14].

Since the $N - \Delta(1232)$ transition dominates at low Q^2, the agreement with the neutron $JLab$ data can be recovered by modifying the low-Q^2 behavior of the asymmetry A_1^Δ. In Fig. 3 the open dots, which nicely fits all the $JLab$ data, correspond to the results of a modified parameterization of g_1^n in which we assume $A_1^\Delta = -0.56 - 0.85\ (Q^2/0.14)e^{-Q^2/0.14}$ instead of the constant value $A_1^\Delta = -0.56$ considered in Ref. [3] and suggested by all multipole analyses (cf. Ref. [14] and references therein). The modification of A_1^Δ has a direct impact also on our parameterization of g_1^p which should be correspondingly modified. Thus we have recalculated the first moment $\Gamma_1(Q^2)$ and our results are reported in Fig. 4. It can be seen that our modified parameterization of g_1^p predicts the same zero-crossing point of the proton

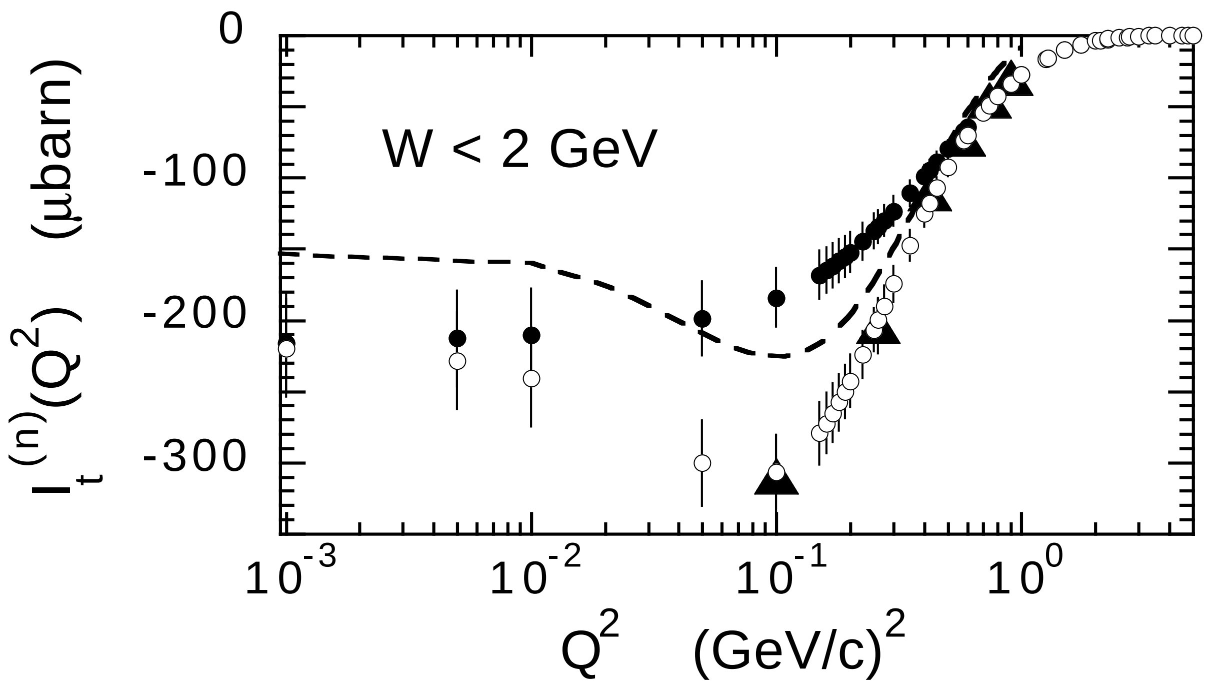

Figure 3: Resonance contribution to the transverse cross section asymmetry of the neutron. Full triangles are data from *JLab* [13]. Open and full dots are our results obtained with and without the low-Q^2 modification of the $N-\Delta(1232)$ transition asymmetry (see text). The dashed line is the result of the unitary isobar model of Ref. [14].

JLab data. Moreover, the low-Q^2 behaviors of $\Gamma_1^{(p)}(Q^2)$ and $\Gamma_1^{(p-n)}(Q^2)$ agree very well with the χpT prediction of Ref. [12].

To sum up, the results we have obtained for the generalized DHG sum rules suggest that more work is needed in order to properly parameterize the low-Q^2 behavior of the $N-\Delta(1232)$ transition.

4 Twist analysis of the proton Nachtmann moments

In this Section we briefly summarize the main results of Ref. [3] concerning the power correction analysis of the polarized proton Nachtmann moments, $M_n^{(1)}(Q^2)$, evaluated in the Q^2-range between $0.5 \div 1$ and 50 $(GeV/c)^2$ using our parameterization of g_1^p and g_2^p. In Ref. [3] the leading twist, $\mu_n^{(1)}(Q^2)$, is treated both at next-to-leading (NLO) order and beyond any fixed order by adopting available soft gluon resummation (SGR) techniques. As for the power corrections, a phenomenological ansätz is considered, viz.

$$M_n^{(1)}(Q^2) = \mu_n^{(1)}(Q^2) + a_n^{(4)} \frac{\mu^2}{Q^2} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(4)}} + a_n^{(6)} \frac{\mu^4}{Q^4} \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right]^{\gamma_n^{(6)}} \tag{4}$$

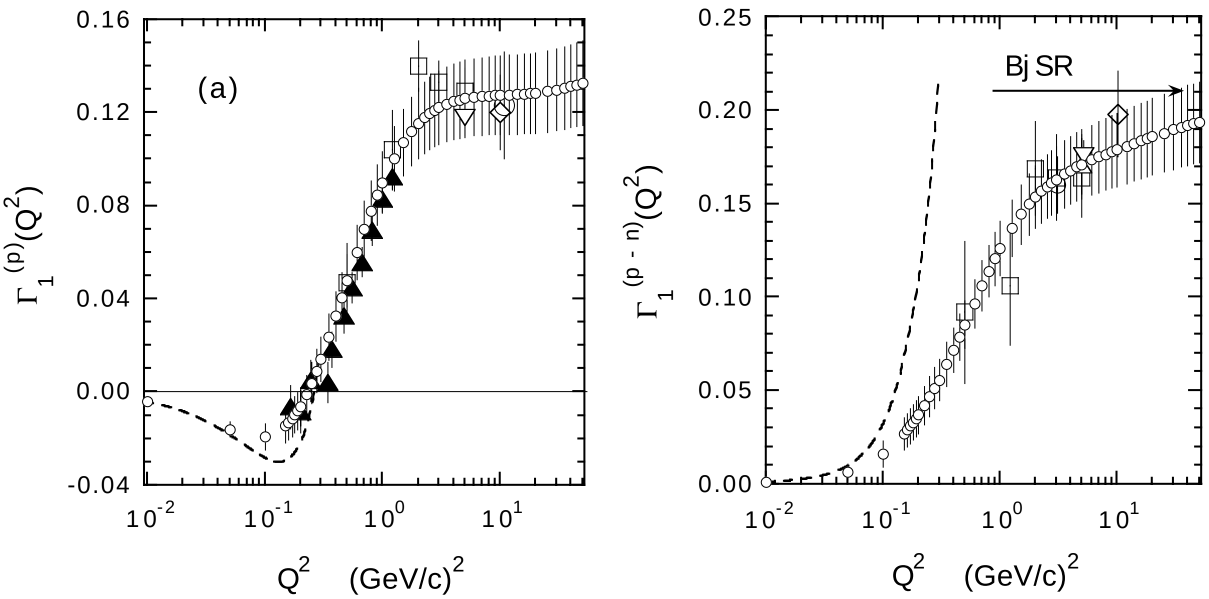

Figure 4: The same as in Fig. 2, with the open dots being our results obtained with the low-Q^2 modification of the $N-\Delta(1232)$ transition asymmetry described in the text. The dashed lines are the χpT predictions of Ref. [12].

where the logarithmic $pQCD$ evolution of the twist-4 (twist-6) contribution is accounted for by an effective anomalous dimension $\gamma_n^{(4)}$ ($\gamma_n^{(6)}$) and the parameter $a_n^{(4)}$ ($a_n^{(6)}$) represents the overall strength of the twist-4 (twist-6) term at the renormalization scale μ^2, chosen to be equal to $\mu^2 = 1$ $(GeV/c)^2$. In order to fix the running of the coupling constant $\alpha_s(Q^2)$, the updated PDG value $\alpha_s(M_Z^2) = 0.118$ is adopted.

As for the first moment ($n = 1$), the leading twist term, $\delta\mu_1^{(1)}(Q^2)$, does not receive any correction from SGR and at NLO it reads as

$$\mu_1^{(1)}(Q^2) = \frac{< e^2 >}{2} \left[\Delta q^{NS} + a_0(Q^2)\right] \left[1 - \frac{\alpha_s(Q^2)}{\pi}\right] \tag{5}$$

The non-singlet moment Δq^{NS} is taken fixed at the value $\Delta q^{NS} = 1.095$, deduced from the experimental values of the triplet and octet axial coupling constants, with the latter obtained under the assumption of $SU(3)$-flavor symmetry. The values of the singlet axial charge $a_0(\mu^2)$ and of the four higher-twist quantities $a_1^{(4)}$, $\gamma_1^{(4)}$, $a_1^{(6)}$ and $\gamma_1^{(6)}$ are determined by fitting our pseudo-data, adopting the least-χ^2 procedure in the Q^2-range between 0.5 and 50 $(GeV/c)^2$. It turns out that the total contribution of the higher twists is tiny for $Q^2 \gtrsim 1$ $(GeV/c)^2$, but it is comparable with the leading twist already at $Q^2 \simeq 0.5$ $(GeV/c)^2$. This means that for g_1^p the onset of *global* duality is expected to occur at $Q^2 \simeq 1$ $(GeV/c)^2$ (cf. Ref. [15] for the case of unpolarized structure functions). In our analysis, where the leading and the higher twists are simultaneously extracted, the singlet axial charge (in the AB scheme) is determined to be $a_0(10\ GeV^2) = 0.16 \pm 0.09$, which nicely agrees with many recent estimates appeared in the literature. Our value of a_0 is therefore significantly below the naive quark-model expectation (i.e. compatible with the well known "proton spin crisis"), but it does not exclude completely a singlet axial charge as large as $\simeq 0.25$.

In case of higher-order moments ($n \geq 3$) both the NLO approximation and the SGR approach have been considered for the leading twist. The comparison of the corresponding twist analyses shows [3] that, except for the third moment, the contribution of the twist-2 is enhanced by soft gluon effects, while the total higher-twist term decreases significantly after the resummation of soft gluons. Thus, as already observed [2] in the unpolarized case, also in the polarized one it is mandatory to go beyond the NLO approximation and to include soft gluon effects in order to achieve a safer extraction of higher twists at large x, particularly for $Q^2 \sim$ few $(GeV/c)^2$.

Finally, the twist decomposition of the polarized Nachtmann moments has been compared with the corresponding one of the unpolarized (transverse) Nachtmann moments obtained in Ref. [2] adopting the same SGR technique. It turns out [3] that the extracted higher-twist contribution appears to be a larger fraction of the leading twist in case of the polarized moments. This findings suggests that spin-dependent multiparton correlations may have more impact than spin-independent ones.

References

[1] G. Ricco *et al.*, *Nucl. Phys.* B **555**, 306 (1999).

[2] S. Simula, *Phys. Lett.* B **493**, 325 (2000).

[3] S. Simula *et al.*, *Phys. Rev.* D **65**, 034017 (2002).

[4] J. Ahrens *et al.*, *Phys. Rev. Lett.* **87**, 022003 (2001).

[5] (a) SMC coll.: B. Adeva *et al.*, *Phys. Rev.* D **58**, 112001 (1998). (b) $E143$ coll.: K. Abe *et al.*, *Phys. Rev.* D **58**, 112003 (1998). (c) $E155$ coll.: P.L. Anthony *et al.*, *Phys. Lett.* B **493**, 19 (2000).

[6] (a) $E142$ coll.: P.L. Anthony *et al.*, *Phys. Rev.* D **54**, 6620 (1996). (b) $E154$ coll.: K. Abe *et al.*, *Phys. Rev. Lett.* **79**, 26 (1997). (c) $HERMES$ coll.: K. Ackerstaff *et al.*, *Phys. Lett.* B **404**, 383 (1997).

[7] Particle Data Group: D.E. Groom *et al.*, *Eur. Phys. Jour.* C **15**, 1 (2000).

[8] A.M. Sandorfi *et al.*, *Phys. Rev.* D **50**, R6681 (1994.

[9] V.D. Burkert, preprint JLAB-PHY-01-19, nucl-ex/0108023.

[10] (a) see [5](b). (b) SMC coll., P. Adams *et al.*: *Phys. Rev.* D **56**, 5330 (1997); see [5](a). (c) EMC coll., J. Ashman *et al.*: *Nucl. Phys.* B **328**, 1 (1989). (d) see [5](c).

[11] X. Ji *et al.*, *Phys. Lett.* B **472**, 1 (2000).

[12] V. Bernard *et al.*, *Phys. Lett.* B **545**, 105 (2002).

[13] *JLab E*94010 coll.: M. Amarian *et al.*, nucl-ex/0205020.

[14] D. Drechsel *et al.*, *Phys. Rev.* D **63**, 114010 (2001).

[15] G. Ricco *et al.*, *Phys. Rev.* C **57**, 356 (1998); *Few-Body Syst. Suppl.* **10**, 423 (1999). S. Simula, *Phys. Lett.* B **481**, 14 (2000); *Phys. Rev.* D **64**, 038301 (2001).